Konishi Anomaly and Central Extension in $\mathcal{N} = \frac{1}{2}$ Supersymmetry

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Abstract

We show that the 4-dimensional $\mathcal{N} = 1/2$ supersymmetry algebra admits central extension. The central charges are supported by domain wall and the central charges are computed. We also determine the Konishi anomaly for $\mathcal{N} = 1/2$ supersymmetric gauge theory. Due to the new couplings in the Lagrangian, many terms appears. We show that these terms sum up to give the expected form for the holomorphic part of the Konishi anomaly. For the anti-holomorphic part, we give a simple argument that the naive generalization has to be modified. We suggest that the anti-holomorphic Konishi anomaly is given by a gauge invariant completion using open Wilson line.

1 Introduction

Quantum field theory on noncommutative space, $[x^\mu, x^\nu] = i\theta^{\mu\nu}$, displays a rich spectrum of unusual properties, some of which are believed to be relevant for quantum gravity [1]. A natural extension of the noncommutative space is to consider deformed superspace. Superspace with only the bosonic coordinates deformed were considered in [2]. The more general superspace where the Grassmann odd part is deformed was also considered in [3]. More recently, starting with the observation of Ooguri and Vafa [4], string theory in graviphoton background has been considered and it is found that a self-dual graviphoton field strength $C_{\mu\nu}$ induces a deformation of the 4-dimensional superspace [4–7] so that the Grassmann odd coordinates become non-anticommutative. In particular, the deformation keeps $\mathcal{N} = 1/2$ supersymmetry [6,7].

Supersymmetric quantum field theory on non-anticommutative superspace was first formulated by Seiberg [6] and deformed $\mathcal{N} = 1/2$ Wess-Zumino model and pure SYM were constructed. Various generalizations are possible. $\mathcal{N} = 1/2$ supersymmetric gauge theory with chiral matters was constructed in [8], where the modification to the supersymmetry transformations of the chiral matter fields were determined. See also [9] for further studies. Nonlinear sigma models (in four or two-dimensions) were considered in [10] and it was found that the non-anticommutative deformation induces in the Lagrangian an infinite number of terms in powers of the auxiliary field. It turns out that the infinite series can be summed up [11–13] and quite remarkably it can be written in terms of a simple smearing of the Zumino's Lagrangian and the holomorphic superpotential [11,12]. Also, gauge theories with extended supersymmetry have been constructed using deformed harmonic superspace [14], and [15] for the deformed $\mathcal{N} = 4$ SYM. Instantons have been studied [16,17].

The above studies are classical. The quantum properties of non-anticommutative supersymmetric theories are interesting and important. Non-anticommutative supersymmetric theories are defined in Euclidean space and are non-hermitian. A priori these theories can have quite different quantum properties from their undeformed cousins due to their different structure in supersymmetry. For the simple $\mathcal{N} = 1/2$ case, it has been argued that the Wess-Zumino model and the supersymmetric gauge theory are renormalizable in the sense that only a finite number of counterterms is needed to be added to the original Lagrangian [18,19]. Some non-renormalization theorems have been argued to remain valid. Also the one loop effective potential of the Wess-Zumino model has been constructed [20]. Further studies of quantum properties of non-anticommutative theories beyond these aspects of renormalizability are however in order.

At the level of supersymmetry algebra, non-anticommutativity modifies the anticommutator of the $\overline{Q}$'s, see (8) below. In standard undeformed supersymmetry, it is well known that at least $\mathcal{N} = 2$ supersymmetry is needed in order to admit a central extension [21,22]. However this holds true when one assumes Lorentz symmetry is unbroken. With Lorentz symmetry broken, one can actually have a central extension in the undeformed $\mathcal{N} = 1$ supersymmetry

algebra. The central charge is carried by a domain wall [23, 24]. For special configuration, the wall is BPS and half of the supersymmetries are left unbroken. It is interesting to see whether the deformed $\mathcal{N} = 1/2$ supersymmetry algebra also admits central extension, and how it is affected by the non-anticommutativity. In section 3, we show that central extension is possible in the $\mathcal{N} = 1/2$ Wess-Zumino model. We construct the domain wall and show that it breaks all the supersymmetries. We also show that the central charge is unaffected by the presence of $C^{\alpha\beta}$.

Another purpose of this paper is to study the quantum properties of 4-dimensional non-anticommutative supersymmetric theories. In section 4, we carry out a detailed analysis of the Konishi anomaly in $\mathcal{N} = 1/2$ supersymmetric gauge theory. This anomaly arises in one-loop. We will find that the holomorphic Konishi anomaly takes the expected form (i.e. dressing up the usual relation with $*$-product), while the anti-holomorphic Konishi anomaly is nontrivially modified. In the next section, we will begin with a brief review of the properties of the $\mathcal{N} = 1/2$ superspace. Discussion of our results and further directions of investigation are given in section 5.

2 $\mathcal{N} = 1/2$ Superspace

Let $(x^\mu, \theta^\alpha, \overline{\theta}^{\dot\alpha})$ be the coordinates of the 4-dimensional non-anticommutative superspace [6]. When a graviphoton background is turned on, the superspace coordinates obey the relations

$$\{\overline{\theta}^{\dot\alpha}, \overline{\theta}^{\dot\beta}\} = \{\overline{\theta}^{\dot\alpha}, \theta^\beta\} = 0, \quad \{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta}, \tag{1}$$

$$[y^\mu, y^\nu] = [y^\mu, \theta^\alpha] = [y^\mu, \overline{\theta}^{\dot\alpha}] = 0, \tag{2}$$

where $y^\mu = x^\mu + i\theta\sigma^\mu\overline{\theta}$ is the chiral coordinate. Functions of θ are Weyl ordered using the $*$-product

$$f(\theta) * g(\theta) = f(\theta) \exp\left(-\frac{C^{\alpha\beta}}{2} \frac{\overleftarrow{\partial}}{\partial\theta^\alpha} \frac{\overrightarrow{\partial}}{\partial\theta^\beta}\right) g(\theta). \tag{3}$$

As is obvious from the above relations, $\overline{\theta}$ is not the complex conjugate of θ. The deformation is possible only for Euclidean space or $(2, 2)$-signature. We will be working in Euclidean space and we follow the convention of [6] to continue to use the Lorentzian signature notation. The $(2, 2)$-signature is relevant for $\mathcal{N} = 2$ string theory and for the studies of non-anticommutative version of supersymmetric integrable systems.

Written in the chiral basis $y, \theta, \overline{\theta}$, the supercharges and covariant derivatives take the stan-

dard expressions

$$Q_\alpha = \frac{\partial}{\partial \theta^\alpha}, \qquad \overline{Q}_{\dot\alpha} = -\frac{\partial}{\partial \overline{\theta}^{\dot\alpha}} + 2i\theta^\alpha \sigma^\mu_{\alpha\dot\alpha} \frac{\partial}{\partial y^\mu}, \tag{4}$$

$$D_a = \frac{\partial}{\partial \theta^\alpha} + 2i\sigma^\mu_{\alpha\dot\alpha} \overline{\theta}^{\dot\alpha} \frac{\partial}{\partial y^\mu}, \quad \overline{D}_{\dot\alpha} = -\frac{\partial}{\partial \overline{\theta}^{\dot\alpha}}. \tag{5}$$

They satisfy

$$\{Q_\alpha, \overline{Q}_{\dot\alpha}\} = 2i\sigma^\mu_{\alpha\dot\alpha} \frac{\partial}{\partial y^\mu}, \tag{6}$$

$$\{Q_\alpha, Q_\beta\} = 0, \tag{7}$$

$$\{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\} = -4C^{\alpha\beta} \sigma^\mu_{\alpha\dot\alpha} \sigma^\nu_{\beta\dot\beta} \frac{\partial^2}{\partial y^\mu \partial y^\nu}, \tag{8}$$

and

$$\{D_\alpha, \overline{D}_{\dot\alpha}\} = -2i\sigma^\mu_{\alpha\dot\alpha} \frac{\partial}{\partial y^\mu}, \tag{9}$$

$$\{D_\alpha, D_\beta\} = \{\overline{D}_{\dot\alpha}, \overline{D}_{\dot\beta}\} = 0, \tag{10}$$

with all the remaining anti-commutators equal to zero. Due to the dependence of $\overline{Q}$'s on the non-anticommutative coordinates θ, $\overline{Q}$ is no longer a symmetry of the noncommutative superspace. The $\mathcal{N} = 1/2$ supersymmetry is generated by the unbroken Q's.

Chiral (resp. anti-chiral) superfields are defined by $\overline{D}_{\dot\alpha}\Phi = 0$ (resp. $D_\alpha\overline{\Phi} = 0$) and are given by the expansion:

$$\Phi(y, \theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y), \tag{11}$$

$$\overline{\Phi}(\overline{y}, \overline{\theta}) = \overline{A}(\overline{y}) + \sqrt{2}\overline{\theta\psi}(\overline{y}) + \overline{\theta\theta F}(\overline{y}), \tag{12}$$

where $\overline{y}^\mu = y^\mu - 2i\theta^\alpha \sigma^\mu_{\alpha\dot\alpha} \overline{\theta}^{\dot\alpha}$. In the presence of gauge symmetry, it is more convenient to parametrize the anti-chiral fields slightly differently, see (68), (69) below, so that the component fields have the standard form of gauge transformation.

3 Wess-Zumino Model: Central Charge and Domain Wall

Consider the $\mathcal{N} = 1/2$ Wess-Zumino model,

$$\mathcal{L} = \int d^4\theta \overline{\Phi} * \Phi + \int d^2\theta \mathcal{W}(\Phi) + \int d^2\overline{\theta}\, \overline{\mathcal{W}}(\overline{\Phi}) \tag{13}$$

with superpotential ($\lambda, \overline{\lambda} > 0$),

$$\mathcal{W}(\Phi) = \mu^2\Phi + \frac{m}{2}\Phi * \Phi - \frac{\lambda}{3}\Phi * \Phi * \Phi,$$

$$\overline{\mathcal{W}}(\overline{\Phi}) = \overline{\mu}^2\overline{\Phi} + \frac{\overline{m}}{2}\overline{\Phi} * \overline{\Phi} - \frac{\overline{\lambda}}{3}\overline{\Phi} * \overline{\Phi} * \overline{\Phi}. \tag{14}$$

Without loss of generality, one can take $m = \overline{m} = 0$. We have [6]

$$\mathcal{L} = \mathcal{L}(C = 0) + \frac{1}{3}\lambda \det C\ F^3, \tag{15}$$

where [1]

$$\begin{aligned}\mathcal{L}(C = 0) &= \partial_\mu\overline{A}\partial^\mu A + i\psi^\alpha\partial_{\alpha\dot\alpha}\overline{\psi}^{\dot\alpha} + \overline{F}F + F\mathcal{W}'(A) - \frac{1}{2}\mathcal{W}''(A)\psi\psi\\ &\quad + \overline{F}\,\overline{\mathcal{W}}'(\overline{A}) - \frac{1}{2}\overline{\mathcal{W}}''(\overline{A})\overline{\psi}\overline{\psi}.\end{aligned} \tag{16}$$

The bosonic equations of motion take the form [2]

$$\Box\overline{A} = F\mathcal{W}'', \qquad \Box A = \overline{F}\,\overline{\mathcal{W}}'', \tag{17}$$

$$F + \overline{\mathcal{W}}' = 0, \qquad \overline{F} + \mathcal{W}' + \lambda\det C\ (\overline{\mathcal{W}}')^2 = 0. \tag{18}$$

Note that those for $A, \overline{F}$ are modified by C, while those of $\overline{A}, F$ are left unchanged. The equations of motion for the fermions are also unmodified by C.

The transformations $\delta\Phi := (\xi Q + \overline{\xi}\,\overline{Q}) * \Phi$, $\delta\overline{\Phi} := (\xi Q + \overline{\xi}\,\overline{Q}) * \overline{\Phi}$ translate to that of the component fields as

$$\delta A = \sqrt{2}\xi\psi + \sqrt{2}i\overline{\xi}_{\dot\gamma}(\overline{\sigma}^\mu)^{\dot\gamma\gamma}\epsilon_{\gamma\alpha}C^{\alpha\beta}\partial_\mu\psi_\beta, \tag{19}$$

$$\delta\psi = \sqrt{2}\xi F + 2i\sigma^\mu\overline{\xi}\partial_\mu A, \tag{20}$$

$$\delta F = \sqrt{2}i\overline{\xi}\overline{\sigma}^\mu\partial_\mu\psi, \tag{21}$$

and

$$\delta\overline{A} = \sqrt{2}\overline{\xi}\overline{\psi}, \tag{22}$$

$$\delta\overline{\psi} = \sqrt{2}\overline{\xi}\overline{F} + \sqrt{2}i\overline{\sigma}^\mu\xi\partial_\mu\overline{A}, \tag{23}$$

$$\delta\overline{F} = \sqrt{2}i\xi\sigma^\mu\partial_\mu\overline{\psi}. \tag{24}$$

It is evident that the theory is invariant under the Q-supersymmetry and broken for the $\overline{Q}$-transformation. One can easily work out the conserved supercurrent

$$J^\mu_\beta = \sqrt{2}(\sigma^\nu\overline{\sigma}^\mu\psi)_\beta\partial_\nu\overline{A} - \sqrt{2}i\overline{\mathcal{W}}'(\sigma^\mu\overline{\psi})_\beta, \tag{25}$$

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[1]We normalize $\int d^2\theta\,\theta^2 = \int d^2\overline{\theta}\,\overline{\theta}^2 = 1$.

[2]Here $\Box$ is the (minus) Laplacian in the Euclidean space.

from which the supercharge

$$Q_\alpha = \int d^3x J_\alpha^0 \tag{26}$$

is obtained. Note that the form of the supercurrent and the supercharge are not modified by $C^{\alpha\beta}$. Quantizing the fermions using the equal time anticommutation relation,

$$\{\psi_\alpha(t,\vec{x}), \overline{\psi}_{\dot{\alpha}}(t,\vec{x}')\} = \delta_{\alpha\dot{\alpha}}\delta(\vec{x}-\vec{x}') \tag{27}$$

and keeping carefully the boundary terms, one obtains for the anticommutator of two super-charges

$$\{Q_\alpha, Q_\beta\} = 4i(\vec{\sigma})_{\alpha\beta} \cdot \int d^3x \vec{\nabla}\overline{\mathcal{W}}(\overline{A}). \tag{28}$$

Here $\vec{\sigma}_{\alpha\beta}$ is defined by $\vec{\sigma}_{\alpha\beta} := \vec{\sigma}_{\alpha\dot{\beta}}\epsilon^{\dot{\beta}\beta}$ and is symmetric. Explicitly $\vec{\sigma}_{\alpha\beta} = \{-\tau^3, i\mathbf{1}, \tau^1\}_{\alpha\beta}$.

The right hand side above is the central charge to the unbroken $\mathcal{N} = 1/2$ supersymmetry algebra. It is a surface term which is normally zero. However the expression is nonzero in the presence of a domain wall. The value of the central charge is proportional to the difference between the vacuum expectation values of $\mathcal{W}$ in the two distinct vacua between which the domain wall lies. For a wall lying in the xy-plane, we have

$$\{Q_\alpha, Q_\beta\} = 2i(\tau_1)_{\alpha\beta}\Sigma\mathsf{A}, \tag{29}$$

where A is the area of the wall and

$$\Sigma := 2\overline{\mathcal{W}}(z=\infty) - 2\overline{\mathcal{W}}(z=-\infty) \tag{30}$$

is the central charge per unit area. Hence we have shown that central extension of the $\mathcal{N} = 1/2$ supersymmetry is possible. Note that the result (28) takes the same form as in the undeformed case with $C^{\alpha\beta} = 0$. However since the equations of motion are modified, the domain wall configuration as well as the values of $\overline{W}$ will be modified in general. Our next task is to solve

$$\partial_z^2\overline{A} = \overline{\mathcal{W}}'\mathcal{W}'', \quad \partial_z^2 A = \overline{\mathcal{W}}''[\mathcal{W}' + \lambda\det C(\overline{\mathcal{W}}')^2] \tag{31}$$

for the domain wall.

In the undeformed case $C^{\alpha\beta} = 0$, the equations reduce to the form

$$\partial_z^2\overline{A} = \overline{\mathcal{W}}'\mathcal{W}'', \quad \partial_z^2 A = \mathcal{W}'\overline{\mathcal{W}}'' \tag{32}$$

for a domain wall extending in the z-direction. These second order equations follows from the first order ones

$$\partial_z\overline{A} = e^{i\beta}\mathcal{W}'(A), \quad \partial_z A = e^{-i\beta}\overline{\mathcal{W}}'(\overline{A}), \tag{33}$$

where β is a constant phase factor. Moreover for real

$$A = \overline{A}, \tag{34}$$

the domain wall satisfies a single first order equation [3]

$$\partial_z A = \mathcal{W}'(A). \tag{35}$$

For example, for the superpotential (14) with $\mu = \overline{\mu}, \lambda = \overline{\lambda}$, we have the solution

$$A = \frac{\mu}{\sqrt{\lambda}} \tanh(\mu\sqrt{\lambda}(z - z_0)). \tag{36}$$

This domain wall interpolates between the two different vacua $A = \pm\mu/\sqrt{\lambda}$ and has a central charge

$$\Sigma = \frac{8\mu^3}{3\sqrt{\lambda}}. \tag{37}$$

For the non-anticommutative case, the equation of motion can no longer be reduced to first order form as in (33). Also obviously one cannot impose the reality condition (34) anymore. Thus one has to solve for the second order equations (31) directly. We begin with an analysis of the vacuum configurations. The classical potential energy of the theory is given by

$$
\begin{aligned}
V &= -\overline{F}F - F\mathcal{W}' - \overline{F\mathcal{W}'} - \frac{1}{3}\lambda\det C\ F^3 \\
&= \overline{\mathcal{W}}'[\mathcal{W}' + \frac{1}{3}\lambda\det C(\overline{\mathcal{W}}')^2].
\end{aligned} \tag{38}
$$

The vacuum configurations satisfy $\partial V/\partial A = 0 = \partial V/\partial\overline{A}$ and one has the possibilities: [4]

$$
\begin{aligned}
\text{(i)}: \quad &\overline{\mathcal{W}}' = 0, \quad \mathcal{W}' = 0, & \tag{39} \\
\text{(ii)}: \quad &\mathcal{W}'' = 0, \quad \overline{\mathcal{W}}'' = 0, & \tag{40} \\
\text{(iii)}: \quad &\mathcal{W}'' = 0, \quad \mathcal{W}' + \lambda\det C(\overline{\mathcal{W}}')^2 = 0, & \tag{41}
\end{aligned}
$$

which implies

$$\text{(i)}: \quad A = \pm\frac{\mu}{\sqrt{\lambda}}, \quad \overline{A} = \pm\frac{\overline{\mu}}{\sqrt{\overline{\lambda}}}, \quad \text{and} \quad V = 0, \tag{42}$$

$$\text{(ii)}: \quad A = \overline{A} = 0, \quad \text{and} \quad V = \overline{\mu}^2(\mu^2 + \frac{\lambda\overline{\mu}^4}{3}\det C), \tag{43}$$

$$\text{(iii)}: \quad A = 0, \quad \overline{A}^2 = (\overline{\mu}^2 \pm \sqrt{\frac{\mu^2}{-\lambda\det C}})/\overline{\lambda}, \quad \text{and} \quad V = \mp\frac{2\mu^2}{3}\sqrt{\frac{\mu^2}{-\lambda\det C}}. \tag{44}$$

The case (iii) is possible only if $\det C < 0$, in which cases we can have new vacuum configuration with energies less than zero.

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[3]In this case $e^{i\beta} = \pm 1$. Moreover we can always choose $e^{i\beta} = 1$ by absorbing the sign into z.

[4]Note that in the first reference of [18], it was assumed the quantity $H(A,\overline{A})$ appearing in $V = \overline{\mathcal{W}}'[\mathcal{W}' - H(A,\overline{A})]$ has a nontrivial dependence on A. This is not the case for our V in (38).

Despite their more complicated form, we are able to solve for and write down several explicit solutions of (31). For simplicity, we will take $\mu = \overline{\mu}, \lambda = \overline{\lambda}$ below. Also since A and $\overline{A}$ can vary independently without any reality constraint, we may take $\overline{A}$ to be sitting at the vev while we allow A to vary. This is possible for $\overline{A} = -\mu/\sqrt{\lambda}$ and with A obeying

$$\partial^2 A = 2\mu\sqrt{\lambda}\frac{\partial\mathcal{W}}{\partial A}. \tag{45}$$

This can be integrated to give

$$\partial A = \sqrt{4\mu\sqrt{\lambda}}\sqrt{\mathcal{W} + k} \tag{46}$$

with an integration constant k; or equivalently

$$\int_\infty^A \frac{dA}{\sqrt{4A^3 - g_2 A - g_3}} = -\sqrt{\frac{\mu\lambda^{3/2}}{3}}(z - z_0), \tag{47}$$

where

$$g_2 = \frac{12\mu^2}{\lambda}, \quad g_3 = -\frac{12k}{\lambda}. \tag{48}$$

For $k \neq 0$, the equation (47) has a solution given in terms of the Weierstrass elliptic function

$$A = \wp(-\sqrt{\mu\lambda^{3/2}/3}\,(z - z_0); g_2, g_3) \tag{49}$$

if the parameters g_2, g_3 given in (48) are chosen such that (in particular, a negative k is needed),

$$g_2 = 60 \sum_{n,m\neq 0} \frac{1}{(2n\omega_1 + 2m\omega_3)^4}, \quad g_3 = 140 \sum_{n,m\neq 0} \frac{1}{(2n\omega_1 + 2m\omega_3)^6}, \tag{50}$$

for some half periods ω_1, ω_3. This solution is singular at $z = z_0$ (and its images) and its physical meaning is not directly clear. For $k = 0$, the equation (46) can be integrated directly and a regular solution can be written down in terms of the Jacobi elliptic function sn,

$$A = \sqrt{\frac{3\mu^2}{\lambda}}\mathrm{sn}^2(\frac{\mu\sqrt{\lambda}}{3^{1/4}}z, i). \tag{51}$$

However this solution is not a domain wall and does not carry a central charge.

We are interested in the domain wall solution, particularly one which carries a nonvanishing central charge. Due to the complexity of (31), we are not able to construct such solutions explicitly. However their existence is easy to demonstrate. To see this, let us try to solve (31) perturbatively with the expansion parameter $\varepsilon := \det C$. Let

$$A = A_0 + \varepsilon A_1 + \cdots, \tag{52}$$
$$\overline{A} = \overline{A}_0 + \varepsilon \overline{A}_1 + \cdots, \tag{53}$$

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where $A_0 = \overline{A}_0$ is given by (36) and $\cdots$ denotes terms of higher order in ε. Note that $A_0, \overline{A}_0 \to \pm\mu/\sqrt{\lambda}$ as $z \to \pm\infty$. A_1 and $\overline{A}_1$ satisfy

$$\partial^2 \overline{A}_1 = 4\lambda\mu^2 \tanh^2(\mu\sqrt{\lambda}z)\, \overline{A}_1 - 2\lambda\mu^2\text{sech}^2(\mu\sqrt{\lambda}z)\, A_1, \tag{54}$$

$$\partial^2 A_1 = 4\lambda\mu^2 \tanh^2(\mu\sqrt{\lambda}z)\, A_1 - 2\lambda\mu^2\text{sech}^2(\mu\sqrt{\lambda}z)\, \overline{A}_1 - 2\lambda^{3/2}\mu^5 \tanh(\mu\sqrt{\lambda}z)\text{sech}^4(\mu\sqrt{\lambda}z).$$

One can easily show that there exists solutions such that

$$A_1, \overline{A}_1 \sim e^{-2\mu\sqrt{\lambda}|z|}, \quad \text{as } |z| \to \infty, \tag{55}$$

and hence the asymptotic values of $A, \overline{A}$ are not affected. The analysis can be easily extended to the higher orders and we conclude that the system (31) admits a domain wall solution which interpolate between the two vacua $\pm\mu/\sqrt{\lambda}$. This domain wall carries the same central charge (37).

In the undeformed case, domain wall satisfying (33) is BPS saturated and preserve half of the $\mathcal{N} = 1$ supersymmetry. This can be seen easily from the supersymmetry transformations (20) and (23) of ψ and $\overline{\psi}$. In fact for a domain wall extending in the xy directions, if (33) is satisfied, then two of the supersymmetries obeying

$$\overline{\xi}^{\dot{\alpha}} = ie^{i\beta}(\sigma^3)^{\dot{\alpha}\alpha}\xi_\alpha, \tag{56}$$

are preserved, that is, a linear combination of the Q and $\overline{Q}$ supersymmetries is preserved. Because of the preserved supersymmetry, the 3-dimensional field theory on the domain wall has vanishing vacuum energy and thus the domain wall energy density is not renormalized. This property does not hold for the the deformed case. In fact in this case, the set of equations (33) are no longer consistent with the equation of motion (31). Therefore the domain wall annihilates all the supersymmetry. This is to be expected since all the $\overline{Q}$'s are broken in the $\mathcal{N} = 1/2$ supersymmetry and this is the reason why the equation of motion cannot be reduced to the first order form (33).

4 $\mathcal{N} = 1/2$ Gauge Theories: Konishi Anomaly and Central Charge

Let us now discuss the case of $\mathcal{N} = 1/2$ gauge theory. In [6]. it is shown that the vector superfield V may be modified with an additional C-dependent part such that the component fields transform canonically under gauge transformation. In the Wess-Zumino gauge, V is given by

$$V(y, \theta, \bar{\theta}) = -\theta\sigma^\mu\overline{\theta}A_\mu(y) + i\theta\theta\overline{\theta}\overline{\lambda}(y) - i\overline{\theta}\overline{\theta}\theta^\alpha \left(\lambda_\alpha(y) + \frac{1}{4}\epsilon_{\alpha\beta}C^{\beta\gamma}\sigma^\mu_{\gamma\dot{\gamma}} \left\{ \overline{\lambda}^{\dot{\gamma}}, A_\mu \right\}(y) \right)$$

$$+ \frac{1}{2}\theta\theta\overline{\theta}\overline{\theta} \left(D(y) - i\partial_\mu A^\mu(y) \right). \tag{57}$$

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V transforms under gauge transformation as [5]

$$e^V \rightarrow e^{-i\overline{\Lambda}} * e^V * e^{i\Lambda}. \tag{58}$$

The gauge transformation which preserves the gauge (57) is given by

$$
\begin{aligned}
\Lambda(y, \theta) &= -\varphi(y), \\
\overline{\Lambda}(\overline{y}, \overline{\theta}) &= -\varphi(\overline{y}) - \frac{i}{2}\overline{\theta\theta}C^{\mu\nu}\{\partial_\mu\varphi, A_\nu\}(\overline{y}) \\
&= -\varphi(y) + 2i\theta\sigma^\mu\overline{\theta}\partial_\mu\varphi(y) - \theta\theta\overline{\theta\theta}\partial^2\varphi(y) - \frac{i}{2}\overline{\theta\theta}C^{\mu\nu}\{\partial_\mu\varphi, A_\nu\},
\end{aligned} \tag{59}
$$

and the gauge transformation of the component fields are the standard ones:

$$
\begin{aligned}
\delta A_\mu = -2\partial_\mu\varphi + i[\varphi, A_\mu], &\qquad \delta D = i[\varphi, D], \\
\delta\overline{\lambda} = i[\varphi, \overline{\lambda}], &\qquad \delta\lambda = i[\varphi, \lambda].
\end{aligned} \tag{60}
$$

Note that although the C-dependent part in (57) and (59) does not take value in the Lie-algebra, nevertheless the component fields transform correctly. The chiral and antichiral field strength superfields are defined by

$$W_\alpha = -\frac{1}{4}\overline{DD}e^{-V}D_\alpha e^V, \quad \overline{W}_{\dot\alpha} = \frac{1}{4}DDe^V\overline{D}_{\dot\alpha}e^{-V} \tag{61}$$

and transform as

$$W_\alpha \rightarrow e^{-i\Lambda} * W_\alpha * e^{i\Lambda}, \quad \overline{W}_{\dot\alpha} \rightarrow e^{-i\overline{\Lambda}} * \overline{W}_{\dot\alpha} * e^{i\overline{\Lambda}}. \tag{62}$$

In terms of components, we have

$$W_\alpha = W_\alpha(C = 0) + \frac{1}{2}C_{\mu\nu}\sigma^{\mu\nu\,\beta}_\alpha\theta_\beta\overline{\lambda\lambda}(y), \tag{63}$$

$$\overline{W}_{\dot\alpha} = \overline{W}_{\dot\alpha}(C = 0) - \theta\theta\left[\frac{C^{\mu\nu}}{2}\{F_{\mu\nu}, \overline{\lambda}_{\dot\alpha}\} + C^{\mu\nu}\{A_\nu, \mathcal{D}_\mu\overline{\lambda}_{\dot\alpha} - \frac{i}{4}[A_\mu, \overline{\lambda}_{\dot\alpha}]\} + \frac{i}{16}|C|^2\{\overline{\lambda\lambda}, \overline{\lambda}_{\dot\alpha}\}\right],$$

where

$$W_\alpha(C = 0) = -i\lambda_\alpha(y) + \left[\delta^\beta_\alpha D(y) - i\sigma^{\mu\nu\,\beta}_\alpha F_{\mu\nu}(y)\right]\theta_\beta + \theta\theta\sigma^\mu_{\alpha\dot\alpha}\mathcal{D}_\mu\overline{\lambda}^{\dot\alpha}(y), \tag{64}$$

$$\overline{W}^{\dot\alpha}(C = 0) = i\overline{\lambda}^{\dot\alpha} + \left[\delta^{\dot\alpha}_{\dot\beta}D - i(\overline{\sigma}^{\mu\nu})^{\dot\alpha}{}_{\dot\beta}F_{\mu\nu}\right]\theta^{\dot\beta} - \overline{\theta\theta}(\overline{\sigma}^\mu)^{\dot\alpha\beta}\mathcal{D}_\mu\lambda_\beta. \tag{65}$$

A general supersymmetric gauge theory will also has matter fields represented by chiral and antichiral superfields. Chiral superfield has the standard component expansion (11). For antichiral field, it is most convenient to parametrize its component expansion in such a way that the component fields transform with standard gauge transformation. The precise form will generally depends on the representation with respect to the gauge group. For example, let S, T

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[5]The exponential functions $e^V = \sum V^{*n}/n!$ is defined with $*$-product.

be chiral superfield in the fundamental and anti-fundamental representation of the gauge group, and $\overline{S}, \overline{T}$ be antichiral superfields in the anti-fundamental and fundamental representations correspondingly

$$S \rightarrow e^{-i\Lambda} * S, \qquad \overline{S} \rightarrow \overline{S} * e^{i\overline{\Lambda}}, \tag{66}$$

$$T \rightarrow T * e^{i\Lambda}, \qquad \overline{T} \rightarrow e^{-i\overline{\Lambda}} * \overline{T}. \tag{67}$$

If we parametrize the $\overline{\theta}\overline{\theta}$ component of $\overline{S}, \overline{T}$ in the following manner [8],

$$\overline{S}(\overline{y}, \overline{\theta}) = \overline{A}_{\mathrm{s}}(\overline{y}) + \sqrt{2}\,\overline{\theta}\overline{\psi}_{\mathrm{s}}(\overline{y}) + \overline{\theta}\overline{\theta} \left(\overline{F}_{\mathrm{s}}(\overline{y}) + iC^{\mu\nu}\partial_\mu(\overline{A}_{\mathrm{s}}A_\nu)(\overline{y}) - \frac{1}{4}C^{\mu\nu}\overline{A}_{\mathrm{s}}A_\mu A_\nu(\overline{y}) \right), \tag{68}$$

$$\overline{T}(\overline{y}, \overline{\theta}) = \overline{A}_{\mathrm{T}}(\overline{y}) + \sqrt{2}\,\overline{\theta}\overline{\psi}_{\mathrm{T}}(\overline{y}) + \overline{\theta}\overline{\theta} \left(\overline{F}_{\mathrm{T}}(\overline{y}) + iC^{\mu\nu}\partial_\mu(A_\nu\overline{A}_{\mathrm{T}})(\overline{y}) - \frac{1}{4}C^{\mu\nu}A_\mu A_\nu \overline{A}_{\mathrm{T}}(\overline{y}) \right), \tag{69}$$

then the component fields of $S, T, \overline{S}, \overline{T}$ all have the standard gauge transformations

$$\begin{aligned} \delta f = i\varphi f, \quad & \text{for} \quad f = A_{\mathrm{s}}, \psi_{\mathrm{s}}, F_{\mathrm{s}}, \overline{A}_{\mathrm{T}}, \overline{\psi}_{\mathrm{T}}, \overline{F}_{\mathrm{T}} \\ \delta f = -if\varphi, \quad & \text{for} \quad f = A_{\mathrm{T}}, \psi_{\mathrm{T}}, F_{\mathrm{T}}, \overline{A}_{\mathrm{s}}, \overline{\psi}_{\mathrm{s}}, \overline{F}_{\mathrm{s}}. \end{aligned} \tag{70}$$

As we shall see later, the form of the supersymmetry transformation (84) of λ_α imposes that the gauge group has to be $U(N)$. The supersymmetry transformation (84) may be modified to adapt for the case of $SU(N)$ [19], however the $\mathcal{N} = 1/2$ superfield formulation requires further investigation. For simplicity, we will take the gauge group to be $U(N)$ in this paper.

4.1 SQCD

A theory of particular interest is the SQCD with $U(N)$ gauge group and N_{f} flavors, with each flavor consisting of a pair of chiral superfields $\{S_{\mathrm{i}}, T_{\mathrm{i}}\}$ in the fundamental and anti-fundamental representations of the gauge group. The superpotential consists of the mass term

$$\mathcal{W}_m = \sum_{\mathrm{i}=1}^{N_{\mathrm{f}}} m_{\mathrm{i}} T_{\mathrm{i}} S_{\mathrm{i}} \tag{71}$$

plus matter self-interaction terms. Without loss of generality, let us consider the case of a single flavor $\{S, T\}$. The $\mathcal{N} = 1/2$ SQCD is given by the Lagrangian

$$\begin{aligned} \mathcal{L} &= \frac{1}{16kg^2} \left(\int d^2\theta \, \mathrm{tr} W^\alpha * W_\alpha + \int d^2\overline{\theta} \, \mathrm{tr}\overline{W}_{\dot\alpha} * \overline{W}^{\dot\alpha} \right) + \int d^4\theta \, (\overline{S} * e^V * S + T * e^{-V} * \overline{T}) \\ &\quad + \int d^2\theta \, mT * S + \int d^2\overline{\theta} \, m\overline{S} * \overline{T}, \end{aligned} \tag{72}$$

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where k is the normalization of the Lie algebra generators: $\text{tr}(T^a T^b) = k\delta^{ab}$. In terms of the component fields, the Lagrangian reads (up to total derivatives)

$$
\begin{aligned}
\mathcal{L} = \ & \frac{1}{16kg^2}\text{tr}\left(-4i\overline{\lambda}\overline{\sigma}^\mu \mathcal{D}_\mu\lambda - F^{\mu\nu}F_{\mu\nu} + \frac{i}{2}F^{\mu\nu}F^{\rho\sigma}\epsilon_{\mu\nu\rho\sigma} + 2D^2 - 2iC^{\mu\nu}F_{\mu\nu}\overline{\lambda}\overline{\lambda} + \frac{|C|^2}{2}(\overline{\lambda}\overline{\lambda})^2\right) \\
& + \overline{F}_{\text{S}}F_{\text{S}} - i\overline{\psi}_{\text{S}}\overline{\sigma}^\mu\mathcal{D}_\mu\psi_{\text{S}} - \mathcal{D}_\mu\overline{A}_{\text{S}}\mathcal{D}^\mu A_{\text{S}} + \frac{1}{2}\overline{A}_{\text{S}}DA_{\text{S}} + \frac{i}{\sqrt{2}}(\overline{A}_{\text{S}}\lambda\psi_{\text{S}} - \overline{\psi}_{\text{S}}\overline{\lambda}A_{\text{S}}) \\
& + \frac{i}{2}C^{\mu\nu}\overline{A}_{\text{S}}F_{\mu\nu}F_{\text{S}} - \frac{\sqrt{2}}{2}C^{\alpha\beta}\sigma^\mu_{\alpha\dot\alpha}\mathcal{D}_\mu\overline{A}_{\text{S}}\,\overline{\lambda}^{\dot\alpha}\psi_{\text{S}\beta} - \frac{|C|^2}{16}\overline{A}_{\text{S}}\overline{\lambda}\overline{\lambda}F_{\text{S}} \\
& + F_{\text{T}}\overline{F}_{\text{T}} - i\psi_{\text{T}}\sigma^\mu\mathcal{D}_\mu\overline{\psi}_{\text{T}} - \mathcal{D}_\mu A_{\text{T}}\mathcal{D}^\mu\overline{A}_{\text{T}} - \frac{1}{2}A_{\text{T}}D\overline{A}_{\text{T}} - \frac{i}{\sqrt{2}}(\psi_{\text{T}}\lambda\overline{A}_{\text{T}} - A_{\text{T}}\overline{\lambda}\,\overline{\psi}_{\text{T}}) \\
& + \frac{i}{2}C^{\mu\nu}F_{\text{T}}F_{\mu\nu}\overline{A}_{\text{T}} - \frac{\sqrt{2}}{2}C^{\alpha\beta}\sigma^\mu_{\alpha\dot\alpha}\psi_{\text{T}\beta}\overline{\lambda}^{\dot\alpha}\mathcal{D}_\mu\overline{A}_{\text{T}} - \frac{|C|^2}{16}F_{\text{T}}\overline{\lambda}\overline{\lambda}\overline{A}_{\text{T}} \\
& + m\left[F_{\text{T}}A_{\text{S}} + A_{\text{T}}F_{\text{S}} - \psi_{\text{T}}\psi_{\text{S}} - \overline{\psi}_{\text{S}}\overline{\psi}_{\text{T}} + \left(\overline{F}_{\text{S}} + iC^{\mu\nu}\partial_\mu(\overline{A}_{\text{S}}A_\nu) - \frac{1}{4}C^{\mu\nu}\overline{A}_{\text{S}}A_\mu A_\nu\right)\overline{A}_{\text{T}}\right. \\
& \left. + \overline{A}_{\text{S}}\left(\overline{F}_{\text{T}} + iC^{\mu\nu}\partial_\mu(A_\nu\overline{A}_{\text{T}}) - \frac{1}{4}C^{\mu\nu}A_\mu A_\nu\overline{A}_{\text{T}}\right)\right],
\end{aligned}
\tag{73}
$$

where

$$
\mathcal{D}_\mu\lambda = \partial_\mu\lambda + \frac{i}{2}[A_\mu,\lambda], \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \frac{i}{2}[A_\mu,A_\nu],
\tag{74}
$$

$$
\mathcal{D}_\mu\psi_{\text{S}} = \partial_\mu\psi_{\text{S}} + \frac{i}{2}A_\mu\psi_{\text{S}}, \quad \mathcal{D}_\mu A_{\text{S}} = \partial_\mu A_{\text{S}} + \frac{i}{2}A_\mu A_{\text{S}},
\tag{75}
$$

$$
\mathcal{D}_\mu\psi_{\text{T}} = \partial_\mu\psi_{\text{T}} - \frac{i}{2}\psi_{\text{T}}A_\mu, \quad \mathcal{D}_\mu A_{\text{T}} = \partial_\mu A_{\text{T}} - \frac{i}{2}A_{\text{T}}A_\mu.
\tag{76}
$$

Note that since the second term of (72) transforms as $\text{tr}\,\overline{W}_{\dot\alpha}*\overline{W}^{\dot\alpha} \to \text{tr}(e^{-i\overline{\Lambda}}*\overline{W}_{\dot\alpha}*\overline{W}^{\dot\alpha}*e^{i\overline{\Lambda}})$ and the $*$-product of $\overline{\Lambda}$ with the rest cannot be ignored, the gauge invariance of $\mathcal{L}$ in the superfield form (72) is not apparent. Nevertheless this term is gauge invariant up to a total derivative as it is clear from the component expression.

The $\mathcal{N} = 1/2$ supersymmetry transformation is given by

$$
\delta_\xi A_\mu = -i\overline{\lambda}\overline{\sigma}_\mu\xi,
\tag{77}
$$

$$
\delta_\xi\lambda_\alpha = i\xi_\alpha D + (\sigma^{\mu\nu}\xi)_\alpha\left(F_{\mu\nu} + \frac{i}{2}C_{\mu\nu}\overline{\lambda}\overline{\lambda}\right), \quad \delta_\xi\overline{\lambda}_{\dot\alpha} = 0,
\tag{78}
$$

$$
\delta_\xi D = -\xi\sigma^\mu\mathcal{D}_\mu\overline{\lambda},
\tag{79}
$$

11

for the gauge multiplet, and [8]

$$\delta_\xi A_{\rm i} = \sqrt{2}\xi\psi_{\rm i}, \quad \delta_\xi \overline{A}_{\rm i} = 0, \tag{80}$$

$$\delta_\xi \psi_{{\rm i}\alpha} = \sqrt{2}\xi_\alpha F_{\rm i}, \quad \delta_\xi \overline{\psi}_{{\rm i}\dot\alpha} = -i\sqrt{2}\mathcal{D}_\mu \overline{A}_{\rm i}(\xi\sigma^\mu)_{\dot\alpha}, \tag{81}$$

$$\delta_\xi F_{\rm i} = 0, \tag{82}$$

$$\delta_\xi \overline{F}_{\rm s} = -i\sqrt{2}\mathcal{D}_\mu \overline{\psi}_{\rm s}\overline{\sigma}^\mu\xi - i\overline{A}_{\rm s}\xi\lambda + C^{\mu\nu}\left\{ \partial_\mu\left(\overline{A}_{\rm s}\xi\sigma_\nu\overline{\lambda}\right) - \frac{i}{2}\left(\overline{A}_{\rm s}\xi\sigma_\nu\overline{\lambda}\right)A_\mu \right\}, \tag{83}$$

$$\delta_\xi \overline{F}_{\rm T} = -i\sqrt{2}\mathcal{D}_\mu \overline{\psi}_{\rm T}\overline{\sigma}^\mu\xi + i\xi\lambda\overline{A}_{\rm T} + C^{\mu\nu}\left\{ \partial_\mu\left(\xi\sigma_\nu\overline{\lambda}\,\overline{A}_{\rm T}\right) + \frac{i}{2}A_\mu\left(\xi\sigma_\nu\overline{\lambda}\,\overline{A}_{\rm T}\right) \right\} \tag{84}$$

for the matters (i = s, T). Note that in the transformation (78) for λ_α, $\overline{\lambda}\lambda$ is given by an anticommutator of the Lie algebra, therefore the transformation for λ is well defined only for $U(N)$.

4.2   Konishi Anomaly

Let us first review the undeformed case. Let S be any of the chiral superfields of the SQCD. Many years ago, it was realized [25–27] that the kinetic term of the chiral matter superfield

$$\mathcal{K} := \overline{S}e^V S \tag{85}$$

satisfies the anomalous equations [6]

$$\frac{1}{4}\overline{D}^2\mathcal{K} = \frac{\partial\mathcal{W}}{\partial S}S + \frac{1}{64\pi^2}\,{\rm tr}(W^\alpha W_\alpha), \tag{87}$$

$$\frac{1}{4}D^2\mathcal{K} = \overline{S}\frac{\partial\overline{\mathcal{W}}}{\partial\overline{S}} + \frac{1}{64\pi^2}\,{\rm tr}(\overline{W}_{\dot\alpha}\overline{W}^{\dot\alpha}). \tag{88}$$

The first piece on the right hand side of (87), (88) is classical. It follows from the classical invariance of the partition function of the theory under the infinitesimal rescaling of S or $\overline{S}$ (all other fields kept fixed)

$$\begin{aligned} S &\to S\epsilon & \text{for (87)}, \\ \overline{S} &\to \overline{\epsilon}\overline{S} & \text{for (88)}, \end{aligned} \tag{89}$$

---

[6]This result applies for more general chiral or nonchiral supersymmetric gauge theory. For a chiral matter S in representation R, one has

$$\frac{1}{4}\overline{D}^2\mathcal{K} = {\rm tr}\left(\frac{\partial\mathcal{W}}{\partial S}S\right) + \frac{T(R)}{64\pi^2}\,{\rm tr}(W^\alpha W_\alpha). \tag{86}$$

The factor $1/64\pi^2$ is with respect to the normalization of the gauge multiplet in the Lagrangian (72), (73). For the "particle physics normalization" $1/4F^{\mu\nu}F_{\mu\nu}$, we have to replace $1/64\pi^2 \to kg^2/16\pi^2$.

$$\underline{\psi_{\text{T}} \qquad\qquad\qquad \psi_{\text{S}}} \quad = \frac{m}{p^2 + m^2}$$

$$\underline{\overline{\psi}_{\text{S}} \qquad\qquad\qquad \overline{\psi}_{\text{T}}} \quad = \frac{m}{p^2 + m^2}$$

$$\underline{\overline{\psi}_{\text{S}} \qquad\qquad\qquad \psi_{\text{S}}} \quad = \frac{\overline{\sigma}^\mu p_\mu}{p^2 + m^2}$$

$$\underline{\psi_{\text{T}} \qquad\qquad\qquad \overline{\psi}_{\text{T}}} \quad = \frac{\sigma^\mu p_\mu}{p^2 + m^2}$$

Figure 1: Contractions for fermions in SQCD.

where ϵ (resp. $\bar{\epsilon}$) is an arbitrary chiral (resp. anti-chiral) superfield. The second piece has its origin in the UV infinities which plague the composite operator $\mathcal{K}$. It is referred to as the Konishi anomaly and is of quantum origin. For example in (87), there are UV divergences in the operator product $\overline{F}_{\text{S}} A_{\text{S}} = -m A_{\text{T}} A_{\text{S}}$, which appears in the $\theta = 0$ component of $-\frac{1}{4}\overline{D}^2\mathcal{K}$ (see (97) below). The UV divergences can be regulated. However additional contributions are induced after the regulators are removed.

The Konishi anomaly can be computed by the standard techniques, such as point splitting method, Pauli-Villars regularization or calculating the anomalous variation of the functional measure. In nonchiral theory like the SQCD we consider here, the Konishi anomaly can be most readily seen by using the Pauli-Villars regularization method. The Pauli-Villars regulator fields consist of a pair of chiral superfields $\mathcal{Q}, \mathcal{R}$ of mass M in the fundamental and anti-fundamental representations. As a result, the operator product $m A_{\text{T}} A_{\text{S}}$ is replaced by

$$m A_{\text{T}} A_{\text{S}} \rightarrow m A_{\text{T}} A_{\text{S}} - M A_{\mathcal{R}} A_{\mathcal{Q}}. \tag{90}$$

The Lagrangian is given by eqn. (72) with m replaced by M (and with $C = 0$ for the undeformed case). The contractions of the fermion fields are given by the Feynman rules in figure 1. They can be obtained by first changing to a basis which diagonalizes the Lagrangian of the fermions. Integrating out the regulator fields in the gluino background, one can show that an anomalous contribution arises from the triangular diagram shown in figure 2. In the limit $M \to \infty$, we have

$$\lim_{M \to \infty} \overline{F}_{\mathcal{Q}} A_{\mathcal{Q}} = \frac{1}{64\pi^2}\text{tr}(\lambda\lambda). \tag{91}$$

Similarly one can consider the other components of the superfield $\frac{1}{4}\overline{D}^2\mathcal{K}$ and find that $\text{tr}(\lambda\lambda)$ is completed to $\text{tr}(W^\alpha W_\alpha)$. Hence the result (87) is obtained. One can establish (88) in the same manner.
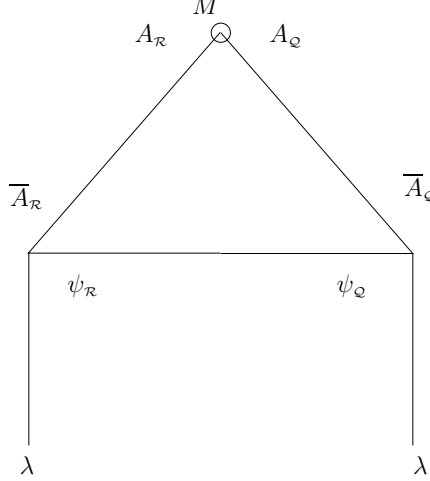
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Figure 2: Diagram contributing to the lowest component of the Konishi anomaly in the unde-formed case.

In the $\mathcal{N} = 1/2$ case, one considers the $*$-gauge invariant operator

$$\mathcal{K} := \overline{S} * e^V * S. \tag{92}$$

From the classical invariance of the partition function of the theory under the variation

$$S \to S * \epsilon, \quad \overline{S} \to \bar{\epsilon} * \overline{S}, \tag{93}$$

one obtains immediately

$$\frac{1}{4}\overline{D}^2\mathcal{K} = \frac{\partial \mathcal{W}}{\partial S} * S, \tag{94}$$

$$\frac{1}{4}D^2\mathcal{K} = \overline{S} * \frac{\partial \overline{\mathcal{W}}}{\partial \overline{S}}. \tag{95}$$

Next we compute the quantum Konishi anomaly. Let us first consider $\frac{1}{4}\overline{D}^2\mathcal{K}$. To do this, we need to first write down the component expansion of $\frac{1}{4}\overline{D}^2\mathcal{K}$ and then determine which composite operators get anomalous contribution. The explicit expansion of $\mathcal{K}$ in components is given in the appendix. It follows that

$$\frac{1}{4}\overline{D}^2\mathcal{K} = \frac{1}{4}\overline{D}^2\mathcal{K}_0 + \frac{1}{4}\overline{D}^2\mathcal{E}, \tag{96}$$

where

$$\frac{1}{4}\overline{D}^2\mathcal{K}_0 = \theta^2\left[\overline{F}_{\mathrm{s}}F_{\mathrm{s}} + (\mathcal{D}_\mu\mathcal{D}^\mu\overline{A}_{\mathrm{s}} - \frac{i}{\sqrt{2}}\overline{\psi}_{\mathrm{s}}\overline{\lambda} + \frac{1}{2}\overline{A}_{\mathrm{s}}D)A_{\mathrm{s}} + i\mathcal{D}_\mu\overline{\psi}_{\mathrm{s}}\overline{\sigma}^\mu\psi_{\mathrm{s}} + \frac{i}{\sqrt{2}}\overline{A}_{\mathrm{s}}\lambda\psi_{\mathrm{s}}\right]$$
$$+ \theta^\alpha\left[\sqrt{2}\overline{F}_{\mathrm{s}}\psi_{\mathrm{s}\alpha} + \sqrt{2}i(\sigma^\mu\mathcal{D}_\mu\overline{\psi}_{\mathrm{s}})_\alpha A_{\mathrm{s}} - i\overline{A}_{\mathrm{s}}\lambda_\alpha A_{\mathrm{s}}\right] + \overline{F}_{\mathrm{s}}A_{\mathrm{s}} \tag{97}$$

$$\frac{1}{4}\overline{D}^2\mathcal{E} = \theta^2\mathcal{E}_{22} + \theta^\beta(\mathcal{E}_{12})_\beta + \mathcal{E}_{02}. \tag{98}$$

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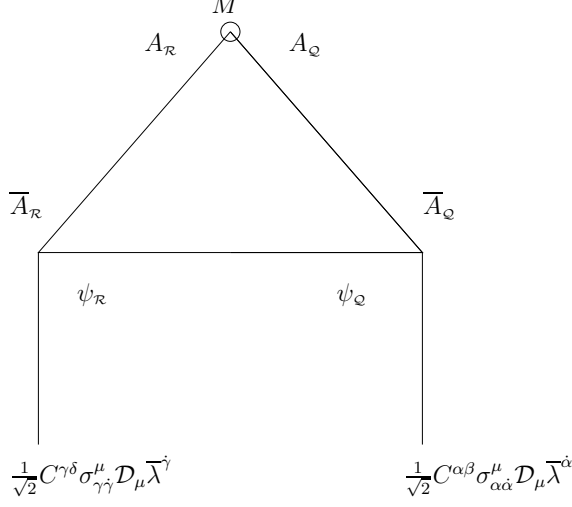Figure 3: Additional contribution to the Konishi anomaly at order θ^0.

Here $\mathcal{E}$ is given by (112) and $\mathcal{E}_{22}, \mathcal{E}_{12}, \mathcal{E}_{02}$ by (113). And

$$F_{\mathrm{S}} = -m\overline{A}_{\mathrm{T}},$$
$$\overline{F}_{\mathrm{S}} = -mA_{\mathrm{T}} - \overline{A}_{\mathrm{S}}(\frac{i}{2}C^{\mu\nu}F_{\mu\nu} - \frac{|C|^2}{16}\overline{\lambda\lambda}). \tag{99}$$

are to be substituted. We remark that one can also apply the equation of motion to each of the components of the θ expansion, (97) and (98), and establishes the result (94) explicitly. It is quite nice that the non-anticommutative $*$-product of the right hand side of (94) is reproduced precisely.

Now let us turn to the computation of the Konishi anomaly. It is natural to guess that the term $\mathrm{tr}(W^\alpha W_\alpha)$ should be completed to $\mathrm{tr}(W^\alpha * W_\alpha)$. We claim that this is indeed the case. The component expansion of $\mathrm{tr}(W^\alpha * W_\alpha)$ can be easily written down,

$$
\begin{aligned}
\mathrm{tr}(W^\alpha * W_\alpha) = & -\mathrm{tr}(\lambda\lambda) - \frac{|C|^2}{4}\mathrm{tr}(\sigma^\mu \mathcal{D}_\mu\overline{\lambda})(\sigma^\nu \mathcal{D}_\nu\overline{\lambda}) \\
& -2i\mathrm{tr}\left[\lambda^\alpha(\delta^\beta_\alpha D - i\sigma^{\mu\nu\beta}_\alpha F_{\mu\nu}) - \lambda_\alpha C^{\alpha\beta}\overline{\lambda\lambda}\right]\theta_\beta \\
& + \left[\mathrm{tr}\left(-2i\overline{\lambda}\overline{\sigma}^\mu\mathcal{D}_\mu\lambda - \frac{1}{2}F_{\mu\nu}F^{\mu\nu} + D^2 + \frac{i}{4}F^{\mu\nu}F^{\rho\sigma}\epsilon_{\mu\nu\rho\sigma}\right)\right. \\
& \left. -iC^{\mu\nu}\mathrm{tr}(F_{\mu\nu}\overline{\lambda\lambda}) + \frac{|C|^2}{4}\mathrm{tr}(\overline{\lambda\lambda})^2\right]\theta^2. \tag{100}
\end{aligned}
$$

Our task is to show that the additional C-dependent terms are precisely generated in (96).

To see this, we have to examine carefully and determine which operators pick up additional anomalous contribution. At level θ^0, we find that only the operator $mA_{\mathrm{T}}A_{\mathrm{S}}$ picks up an

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operator	contributing diagram	coefficient c
$iM\theta_\gamma C^{\gamma\alpha}\overline{A}_\mathcal{Q}\lambda_\alpha\overline{A}_\mathcal{R}$	figure 4a	-1
$-\sqrt{2}MA_\mathcal{R}\theta\psi_\mathcal{Q}$	figure 4b	2/3
$i\overline{\psi}_\mathcal{Q}\overline{\lambda}\theta_\gamma C^{\gamma\alpha}\psi_{\mathcal{Q}\alpha}$	figure 4c	1
$\sqrt{2}\theta_\gamma C^{\gamma\alpha}\square\overline{A}_\mathcal{Q}\psi_{\mathcal{Q}\alpha}$	figure 4d	2/3
$-iM\sqrt{2}\theta_\gamma C^{\gamma\alpha}\epsilon_{\alpha\kappa}(\overline{\sigma}^\mu)^{\dot\alpha\kappa}\partial_\mu\overline{\psi}_{\mathcal{Q}\dot\alpha}\overline{A}_\mathcal{R}$	figure 4e	2/3

Table 1: Operators contributing to Konishi anomaly at order θ. Deformed case.

anomalous contribution. The contributing diagram is shown in figure 3. At level θ, we list in table 1 the operators (of the regulator fields) which contribute. They all contribute to generate the operator

$$c \times \frac{1}{64\pi^2}i\lambda_\alpha C^{\alpha\beta}\theta_\beta\overline{\lambda\lambda} \tag{101}$$

with coefficient c. In total, we obtain $c = 2$, which is precisely what is needed in (100). Finally, at level θ^2, we list in table 2 the operators (of the regulator fields) which contribute [7]. They contribute to generate the operator

$$d_1 \times \frac{1}{64\pi^2}\left(\frac{i}{2}C^{\mu\nu}\mathrm{tr}(F_{\mu\nu}\overline{\lambda\lambda}) - \frac{|C|^2}{16}\mathrm{tr}(\overline{\lambda\lambda})^2\right) + d_2 \times \frac{1}{64\pi^2}\frac{|C|^2}{48}\mathrm{tr}(\overline{\lambda\lambda})^2 \tag{102}$$

with coefficients d_1, d_2. Adding up their contributions, we get

$$\frac{1}{64\pi^2}\left(-iC^{\mu\nu}\mathrm{tr}(F_{\mu\nu}\overline{\lambda\lambda}) + \frac{|C|^2}{4}\mathrm{tr}(\overline{\lambda\lambda})^2\right) \tag{103}$$

which is precisely what is needed in (100). Therefore we obtain the result

$$\frac{1}{4}\overline{D}^2\mathcal{K} = \frac{\partial\mathcal{W}}{\partial S} * S + \frac{1}{64\pi^2}\mathrm{tr}(W^\alpha * W_\alpha). \tag{104}$$

We note that the relation (104) is gauge invariant. This can be seen either by checking the component form, or by noting that the gauge parameter Λ in the gauge transformation (62) is independent of θ and hence insensitive to non-anticommutativity. We also remark that for the undeformed case, it has been argued that the Konishi anomaly satisfies an Adler-Bardeen theorem and is not renormalized beyond 1-loop [29]. It will be interesting to check it for the deformed case. For $\frac{1}{4}D^2\mathcal{K}$, we note that the (anti-holomorphic) Konishi anomaly cannot be simply given by $\mathrm{tr}(\overline{W}*\overline{W})$ as it is not gauge invariant. This is obvious due to the θ-dependence of $\overline{\Lambda}$ and the form of the gauge transformation (62). The gauge non-invariance of $\mathrm{tr}(\overline{W}*\overline{W})$ can also be seen explicitly in the component form. For example at order $\overline{\theta}^2$, $\mathrm{tr}(\overline{W}*\overline{W})$ has a C-dependent part,

$$\mathrm{tr}\left(-iC^{\mu\nu}\mathrm{tr}(F_{\mu\nu}\overline{\lambda\lambda}) + \frac{|C|^2}{4}\mathrm{tr}(\overline{\lambda\lambda})^2\right) - 2iC^{\mu\nu}\partial_\mu\mathrm{tr}(\overline{\lambda\lambda}A_\nu) \tag{105}$$

---

[7]Note that the operator $\overline{F}_\mathrm{S}F_\mathrm{S} + (\frac{i}{2}C^{\mu\nu}\mathrm{tr}(F_{\mu\nu}\overline{\lambda\lambda}) - \frac{|C|^2}{16}\mathrm{tr}(\overline{\lambda\lambda})^2)F_\mathrm{S} = -m^2A_\mathrm{T}\overline{A}_\mathrm{T}$.

operator	contributing diagram	d_1	d_2
$M^2 A_\mathcal{R} \overline{A}_\mathcal{R}$	figure 5a	0	1
$M^2 A_\mathcal{R} \overline{A}_\mathcal{R}$	figure 5b	-1/3	0
$-\frac{i}{\sqrt{2}} \overline{\psi}_\mathcal{Q} \overline{\lambda} A_\mathcal{Q}$	figure 5c	-1	0
$-\frac{i}{\sqrt{2}} \overline{\psi}_\mathcal{Q} \overline{\lambda} A_\mathcal{Q}$	figure 5d	0	4
$i\partial_\mu \overline{\psi}_\mathcal{Q} \overline{\sigma}^\mu \psi_\mathcal{Q}$	figure 5e	0	-3
$i\partial_\mu \overline{\psi}_\mathcal{Q} \overline{\sigma}^\mu \psi_\mathcal{Q}$	figure 5f	-2/3	0
$-\frac{1}{\sqrt{2}} C^{\beta\gamma} \sigma^\mu_{\gamma\dot\gamma} \partial_\mu \overline{A}_\mathcal{Q} \overline{\lambda}^{\dot\gamma} \psi_{\mathcal{Q}\beta}$	figure 5g	0	4

Table 2: Operators contributing to Konishi anomaly at order θ^2. Deformed case.

which is not gauge invariant. A natural guess is that the anti-holomorphic Konishi anomaly is given by the gauge invariant extension of $\mathrm{tr}(\overline{W}_{\dot\alpha} * \overline{W}^{\dot\alpha})$. This is supported by the fact that, apart from the total derivative term, the operators in (105) are indeed generated at one loop by exactly the same set of diagrams in table 2. As in the case of noncommutative gauge theory, it may be possible to obtain the required gauge invariant extension with the help of Wilson line [28]. It is also possible that the Adler-Bardeen theorem for the anti-holomorphic Konishi anomaly does not hold anymore and the higher loops contribute. We leave the investigation of these issues for a further study.

4.3 Central Charge

Let us now consider the central extension in the SQCD (72). To do this, we need to derive the form of the supercharges, or the supercurrent. Under the supersymmetry transformations (77), (80), the Lagrangian (73) changed by a total derivative, from which one can derive the supercurrent

$$
\begin{aligned}
J^\mu_\alpha &= \frac{-i}{4kg^2} (\overline{\lambda}\overline{\sigma}^\mu \sigma^{\rho\nu})_\alpha \left(F_{\rho\nu} + \frac{i}{2} C_{\rho\nu} \overline{\lambda}\lambda \right) \\
&+ \sqrt{2}\mathcal{D}_\nu \overline{A}_\mathrm{S}(\sigma^\nu \overline{\sigma}^\mu \psi_\mathrm{S})_\alpha + \frac{1}{2}\overline{A}_\mathrm{S}(\sigma_\mu \overline{\lambda})_\alpha A_\mathrm{S} + \sqrt{2}(\sigma^\nu \overline{\sigma}^\mu \psi_\mathrm{T})_\alpha \mathcal{D}_\nu \overline{A}_\mathrm{T} + \frac{1}{2}A_\mathrm{T}(\sigma_\mu \overline{\lambda})_\alpha \overline{A}_\mathrm{T} \\
&- m \left(i\sqrt{2}(\sigma^\mu \overline{\psi}_\mathrm{S})_\alpha \overline{A}_\mathrm{T} + i\sqrt{2}\overline{A}_\mathrm{S}(\sigma^\mu \overline{\psi}_\mathrm{T})_\alpha + 2C^{\mu\nu}\overline{A}_\mathrm{S}(\sigma_\nu \overline{\lambda})_\alpha \overline{A}_\mathrm{T} \right).
\end{aligned}
\tag{106}
$$

Note that unlike the case of the Wess-Zumino model, the current is modified by $C^{\alpha\beta}$. The modification is due to the additional terms in the supersymmetry transformations (77)-(84) that are needed in order to keep the WZ gauge. However like the case of the Wess-Zumino model, the form of the commutator $\{Q_\alpha, Q_\beta\}$ is not modified by C. We have

$$
\{Q_\alpha, Q_\beta\} = -4i(\vec{\sigma})_{\alpha\beta} \cdot \int d^3x \vec{\nabla}(m\overline{A}_\mathrm{S}\overline{A}_\mathrm{T}).
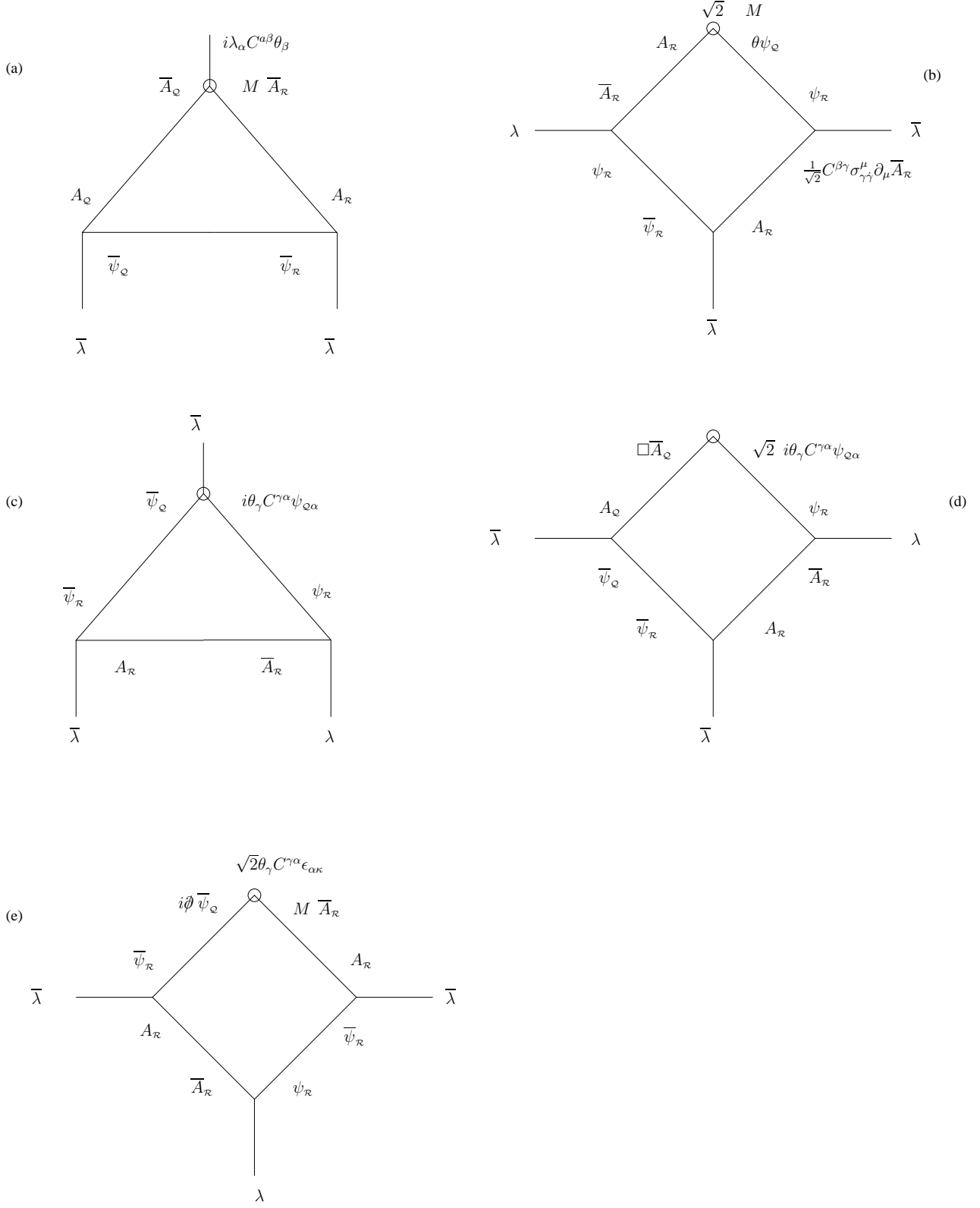\tag{107}
$$

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(a)

$i\lambda_\alpha C^{a\beta}\theta_\beta$

$\overline{A}_\mathcal{Q}$ $M\ \overline{A}_\mathcal{R}$

$A_\mathcal{Q}$ $A_\mathcal{R}$

$\overline{\psi}_\mathcal{Q}$ $\overline{\psi}_\mathcal{R}$

$\overline{\lambda}$ $\overline{\lambda}$

(b)

$\sqrt{2}$ M

$A_\mathcal{R}$ $\theta\psi_\mathcal{Q}$

$\overline{A}_\mathcal{R}$ $\psi_\mathcal{R}$

λ $\overline{\lambda}$

$\psi_\mathcal{R}$ $\frac{1}{\sqrt{2}}C^{\beta\gamma}\sigma^\mu_{\gamma\dot\gamma}\partial_\mu\overline{A}_\mathcal{R}$

$\overline{\psi}_\mathcal{R}$ $A_\mathcal{R}$

$\overline{\lambda}$

(c)

$\overline{\lambda}$

$\overline{\psi}_\mathcal{Q}$ $i\theta_\gamma C^{\gamma\alpha}\psi_{\mathcal{Q}\alpha}$

$\overline{\psi}_\mathcal{R}$ $\psi_\mathcal{R}$

$A_\mathcal{R}$ $\overline{A}_\mathcal{R}$

$\overline{\lambda}$ λ

(d)

$\Box\overline{A}_\mathcal{Q}$ $\sqrt{2}\ i\theta_\gamma C^{\gamma\alpha}\psi_{\mathcal{Q}\alpha}$

$A_\mathcal{Q}$ $\psi_\mathcal{R}$

$\overline{\lambda}$ λ

$\overline{\psi}_\mathcal{Q}$ $\overline{A}_\mathcal{R}$

$\overline{\psi}_\mathcal{R}$ $A_\mathcal{R}$

$\overline{\lambda}$

(e)

$\sqrt{2}\theta_\gamma C^{\gamma\alpha}\epsilon_{\alpha\kappa}$

$i\partial\!\!\!/\,\overline{\psi}_\mathcal{Q}$ $M\ \overline{A}_\mathcal{R}$

$\overline{\psi}_\mathcal{R}$ $A_\mathcal{R}$

$\overline{\lambda}$ $\overline{\lambda}$

$A_\mathcal{R}$ $\overline{\psi}_\mathcal{R}$

$\overline{A}_\mathcal{R}$ $\psi_\mathcal{R}$

λ

Figure 4: Additional contributions to the Konishi anomaly at order θ^1.
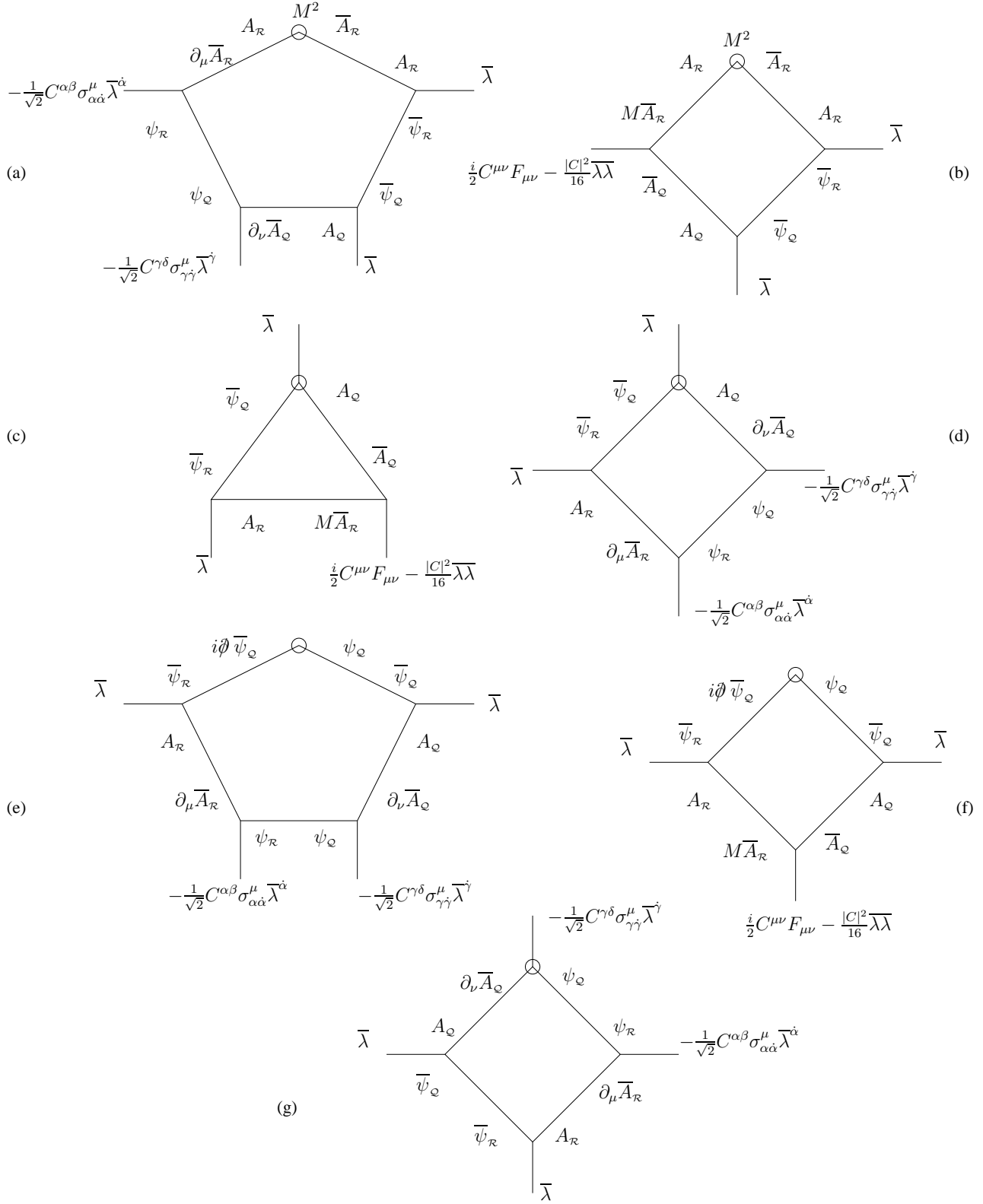
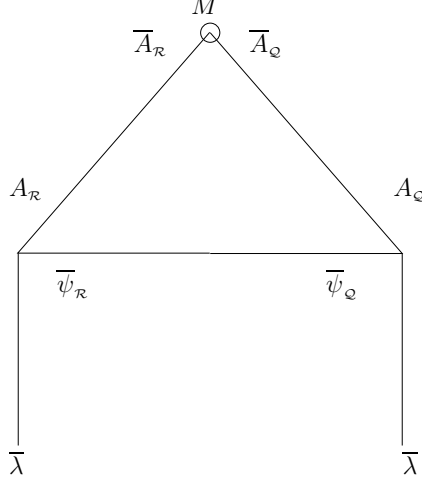Figure 5: Additional contributions to the Konishi anomaly at order θ^2.

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Figure 6: Anomalous contribution to central charge.

The above contribution to the central charge is classical and came from the superpotential $\overline{\mathcal{W}} = m\overline{S} * \overline{T}$. In addition to this classical contribution, there are additional contributions quantum mechanically. First there is a contribution from the usual Konishi anomaly. Indeed the operator $m\overline{A}_S\overline{A}_T$ is the lowest component of $-\frac{1}{4}\overline{D}^2\mathcal{K}$. This operator pick up a quantum contribution [8] from the diagram in figure 6. We obtain

$$m\overline{A}_S\overline{A}_T \rightarrow m\overline{A}_S\overline{A}_T - \frac{1}{64\pi^2}\text{tr}(\overline{\lambda}\overline{\lambda}). \tag{108}$$

In addition to this contribution which has a origin of Konishi anomaly, there is also a contribution from the anomaly in the supercurrent. In the undeformed case, this gives rise to $\frac{N}{64\pi^2}\text{tr}(\overline{\lambda}\overline{\lambda})$. A study of anomaly in the supercurrent multiplet in $\mathcal{N} = 1/2$ gauge theory is in progress and we expect the same contribution to the central charge as in the undeformed case [30]. Assuming this is the case, we obtain the central extension for the $\mathcal{N} = 1/2$ SQCD,

$$\{Q_\alpha, Q_\beta\} = -4i(\vec{\sigma})_{\alpha\beta} \cdot \int d^3x \vec{\nabla}\left(m\overline{A}_S\overline{A}_T - \frac{N - N_f}{64\pi^2}\overline{\lambda}\overline{\lambda}\right). \tag{109}$$

Note that this is the same form as in the undeformed case. Gluino condensate in $\mathcal{N} = 1/2$ gauge theory has been examined in in [17] and it has been found that their values are unmodified by $C^{\alpha\beta}$. However as in the case of the Wess-Zumino model, the value of the central charge may depends on C through the scalar profile.

---

[8]Note that here there is no analogous contribution as the one in figure 3 because the counterpart of the coupling $-\frac{1}{\sqrt{2}}C^{\alpha\beta}\sigma^\mu_{\alpha\dot\alpha}\mathcal{D}_\mu\overline{A}\,\overline{\lambda}^{\dot\alpha}\psi_\beta$ is absent in the Lagrangian (73).

5 Discussion

In this paper, we have shown that it is possible to centrally extend the $\mathcal{N} = 1/2$ supersymmetry algebra and we determine the field theoretic form of the central extension in Wess-Zumino model and $\mathcal{N} = 1/2$ supersymmetric gauge theory. The domain wall we constructed satisfies asymptotically $A = \overline{A}$ and reduces to the standard domain wall solution when $C \to 0$. It has a central charge independent of C. In principle it is possible to construct domain wall with more general asymptotic behaviour (i.e. tending to different vev's). It is interesting to construct such more general domain walls. As the $\mathcal{N} = 1/2$ supersymmetric gauge theory can be constructed as gauge theory on D-brane, it is interesting to understand dynamical aspects such as confinement, mass gap and chiral symmetry breaking, in terms of a D-brane construction [31].

We have also established the form of the holomorphic Konishi anomaly. For the anti-holomorphic one, we show that the naive extension has to be modified and we suggest that the correct form is to be given by a gauge invariant completion of the term $\mathrm{tr}\overline{W} * \overline{W}$. It will be interesting to perform a full analysis of this. Konishi anomaly is related to the anomaly of the supercurrent multiplet. It is also very interesting to determine the structure of the non-anticommutative anomaly supermultiplet.

Konishi anomaly has many physical applications. In this paper we discuss its relation with the central charge of the $\mathcal{N} = 1/2$ supersymmetric gauge theory. We expect that the more general form of the Konishi anomaly [32] will shed light on a deformed version of the Dijkgraff-Vafa theory [33].

Given now the much nicer result for the lower dimensional nonlinear sigma model [11–13], it will be interesting to determine the condition for the vanishing of the one loop beta function and see how the usual Ricci flatness condition is modified by non-anticommutativity.

Acknowledgements

A   Components form of $\mathcal{K}$

In this appendix, we give the components form of $\mathcal{K} = \overline{\Phi} * e^V * \Phi$, where Φ is a chiral superfield (11) and $\overline{\Phi}$ is of the form (68). We have

$$\mathcal{K} = \mathcal{K}_0 + \mathcal{E} \tag{110}$$

where the C-independent part $\mathcal{K}_0$ is

$$
\begin{aligned}
\mathcal{K}_0 \;=\;& \theta^2\bar{\theta}^2\left[\overline{F}F + \mathcal{D}_\mu\mathcal{D}^\mu\overline{A}A + i\mathcal{D}_\mu\overline{\psi}\bar{\sigma}^\mu\psi + \frac{i}{\sqrt{2}}(\overline{A}\lambda\psi - \overline{\psi}\lambda A) + \frac{1}{2}\overline{A}DA\right] \\
&+\bar{\theta}^2\left[\overline{F}A + \sqrt{2}\overline{F}\theta\psi + \sqrt{2}i\theta\sigma^\mu\mathcal{D}_\mu\overline{\psi}A - i\theta\,\overline{A}\lambda A\right] \\
&+\theta^2\left[\overline{A}F + \sqrt{2}\overline{\theta\psi}F - \sqrt{2}i\mathcal{D}_\mu\overline{A}\,\bar{\theta}\sigma^\mu\psi + i\bar{\theta}\,\overline{A}\lambda A\right] \\
&+\overline{A}A + \sqrt{2}\overline{A}\theta\psi + \sqrt{2}\overline{\theta\psi}A + \theta\sigma^\mu\bar{\theta}(-2i\mathcal{D}_\mu\overline{A}A + \overline{\psi}\bar{\sigma}_\mu\psi),
\end{aligned}
\tag{111}
$$

and the C-dependent part $\mathcal{E}$ has the expansion

$$\mathcal{E} = \theta^2\bar{\theta}^2\mathcal{E}_{22} + \theta^\beta\bar{\theta}^2(\mathcal{E}_{12})_\beta + \bar{\theta}^2\mathcal{E}_{02} + \theta_\alpha\bar{\theta}^{\dot\alpha}(\mathcal{E}_{11})^\alpha{}_{\dot\alpha} + \bar{\theta}^{\dot\alpha}(\mathcal{E}_{01})_{\dot\alpha}, \tag{112}$$

where $\mathcal{E}_{mn}$ denotes the coefficient of $(\theta)^m(\bar{\theta})^n$ in $\mathcal{E}$:

$$
\begin{aligned}
\mathcal{E}_{22} \;=\;& \frac{i}{2}C^{\mu\nu}\overline{A}F_{\mu\nu}F - \frac{|C|^2}{16}\overline{A\lambda\lambda}F - \frac{1}{\sqrt{2}}C^{\beta\gamma}\sigma^\mu_{\gamma\dot\gamma}\mathcal{D}_\mu\overline{A}\,\overline{\lambda}^{\dot\gamma}\psi_\beta, \\
(\mathcal{E}_{12})_\beta \;=\;& \sqrt{2}\epsilon_{\beta\gamma}C^{\gamma\alpha}\left[(\mathcal{D}_\mu\mathcal{D}^\mu\overline{A} + \frac{1}{2}\overline{A}D - \frac{i}{\sqrt{2}}\overline{\psi\lambda})\psi_\alpha + \frac{i}{\sqrt{2}}\overline{A}\lambda_\alpha F + i\epsilon_{\alpha\kappa}(\bar{\sigma}^\mu)^{\dot\alpha\kappa}\mathcal{D}_\mu\overline{\psi}_{\dot\alpha}F\right] \\
&+\mathcal{D}_\mu\overline{A}\sigma^\mu_{\gamma\dot\gamma}\overline{\lambda}^{\dot\gamma}C^{\gamma\kappa}(C_{\kappa\beta}F - \epsilon_{\kappa\beta}A) + \frac{i}{\sqrt{2}}C^{\mu\nu}\overline{A}F_{\mu\nu}\psi_\beta - \frac{|C|^2}{8\sqrt{2}}\overline{A\lambda\lambda}\psi_\beta, \\
\mathcal{E}_{02} \;=\;& iC^{\alpha\gamma}\sigma^\mu_{\gamma\dot\alpha}\mathcal{D}_\mu\overline{\psi}^{\dot\alpha}\psi_\alpha + \frac{1}{\sqrt{2}}C^{\alpha\gamma}C_{\gamma\kappa}\epsilon^{\kappa\beta}\sigma^\mu_{\beta\dot\alpha}\mathcal{D}_\mu\overline{A}\overline{\lambda}^{\dot\alpha}\psi_\alpha + \frac{i}{2}C^{\mu\nu}\overline{A}F_{\mu\nu}A \\
&-\frac{|C|^2}{4}\left(\mathcal{D}_\mu\mathcal{D}^\mu\overline{A}\,F + \frac{1}{4}\overline{A\lambda\lambda}A - \frac{i}{\sqrt{2}}\overline{\psi\lambda}F + \frac{1}{2}\overline{A}DF\right) - \frac{i}{\sqrt{2}}\overline{A}\lambda_\alpha C^{\alpha\beta}\psi_\beta, \\
(\mathcal{E}_{11})^\alpha{}_{\dot\alpha} \;=\;& -2iC^{\beta\alpha}\sigma^\mu_{\beta\dot\alpha}\mathcal{D}_\mu\overline{A}\,F + \sqrt{2}i\overline{A}\lambda_{\dot\alpha}C^{\alpha\beta}\psi_\beta, \\
(\mathcal{E}_{01})_{\dot\alpha} \;=\;& i\sqrt{2}\mathcal{D}_\mu\overline{A}\psi_\alpha C^{\alpha\gamma}\sigma^\mu_{\gamma\dot\alpha} + \frac{i}{4}|C|^2\overline{A}\lambda_{\dot\alpha}F.
\end{aligned}
\tag{113}
$$

We have dropped the subscript s for the fields here. Here $C_{\alpha\beta} := \epsilon_{\alpha\alpha'}\epsilon_{\beta\beta'}C^{\alpha'\beta'}$, $C^{\mu\nu} := C^{\alpha\beta}\epsilon_{\beta\gamma}(\sigma^{\mu\nu})_\alpha{}^\gamma$, $|C|^2 := C^{\mu\nu}C_{\mu\nu} = 4\det C$. The Konishi current $T * e^{-V} * \overline{T}$ for chiral field T in anti-fundamental representation can be similarly written down.

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